IN REFERENCE TO A CERTAIN SOLUTION OF THE PROBLEM CONCERNING THE EFFECT OF A VARIABLE-OVER-THE-SURFACE THERMAL FLUX ON THE HEAT TRANSFER IN A TURBULENT STREAM

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In the article by G. S. Ambrok [1] the author shows a "relatively simple solution" of the said problem and gives a calculation formula applicable to "an arbitrary heat load distribution."

Considering that published analytical solutions to the energy equation for an incompressible fluid with constant physical properties and a longitudinally variable density of the thermal flux from walls to fluid is not expressible by simple formulas, any attempt to come up with a less rigorous but simpler solution is of considerable practical interest.

Unfortunately, the solution in [1] is based on an erroneous premise, as will be shown here.

The scheme of the solution in [1] is, indeed, simple. The stream is considered to consist of two layers. In the turbulent mainstream one assumes, according to Prandtl, that

$$q = -\rho C_p \varkappa^2 y^2 \frac{du}{dy} \cdot \frac{dt}{dy}, \tag{1}$$

and in the laminar sublayer

$$q = -\lambda \frac{dt}{dy}. \tag{2}$$

The velocity profile is assumed logarithmic and the thickness of the thermal boundary layer is assumed to follow the relation $\delta_T = \alpha/\mathrm{Pr} \cdot \nu/v_*$ with $v_* = \sqrt{\tau_W/\rho}$. The values of the turbulence parameters are assumed constant and equal to $\varkappa = 0.4$ and $\alpha = 11.5$. In order to obtain the temperature profile from (1) and (2), and then to calculate the stream temperature at the wall and the mean temperature of the stream as well as the Nusselt number, the author of [1] stipulates a linear radial distribution of thermal flux density:

$$q = A_0 + A_1 y. \tag{3}$$

Simple mathematical operations yield an expression for the Nusselt number:

$$\mathrm{Nu} = \frac{0.14\,\mathrm{Pe}\sqrt{\xi}}{\ln(\mathrm{Pe}\sqrt{\xi}) - B\,(0.33 + 85/(\mathrm{Pe}\sqrt{\xi}) - 1.08}, \tag{4}$$

where ξ is the hydraulic drag coefficient, Pe is the Peclet number, and $B = r_0 A_1/A_0$.

Until now we have been dealing with a well known procedure for solving the problem of turbulent heat transfer in a two-layer system. The author of [1] applies next an original method of determining the coefficient A_1 in Eq. (3) (coefficient A_0 is found directly from the condition $q_{y=0} = A_0 = q_W$). For this purpose, the author uses the energy equation in the form

$$C_p \rho u \frac{\partial t}{\partial x} = -\frac{\partial q_x}{\partial x} - \frac{\partial q_y}{\partial y} \tag{5}$$

and asserts that, since at the wall $u = 0$, this equation supposedly yields $[dq_y/dy]_{y=0} = -[dq_W/dx] = A_1$. From this he has $B = -r_0(dq_W/dx)/q_W$, and the term with coefficient B in the denominator of Eq. (4) accounts

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for the effect of the arbitrarily variable component of the heat load at the wall. It is easy to see, however, that the author is in error when he substitutes dq_w/dx for $[dq_x/dx]_{y=0}$. As a matter of fact, $dq_w/dx = [dq_y/dx]_{y=0}$. Therefore, the "justification" with which the author of [1] arrived at his value of A_1 was wrong.

We will now consider what happens if the radial distribution of thermal flux density q is a priori (without any justification) assumed

$$q = q_{\text{ст}} - \frac{dq_{\text{ст}}}{dx} y. \tag{6}$$

Considering the condition at the pipe axis $q_{y=r_0} = 0$, we obtain $dq_w/dx = q_w/r_0$. Integrating with respect to x and determining the integration constants from the condition $q_w|_{x=0} = q_{w_0}$ will yield $q_w = q_{w_0} \exp(x/r_0)$. Coefficient B in formula (4) becomes $B = -1$ and this formula then yields Nu = const.

In the best case, therefore, the analysis in [1] is applicable only to the region of steady heat transfer with an exponential heat load distribution on the wall. As has been shown in several studies, under such a distribution the value of the Nusselt number does, indeed, stabilize within some distance from where the heating begins.

LITERATURE CITED

1. G. S. Ambrok, Inzh.-Fiz. Zh., 4, No. 7 (1961).